circuits which exist when the jig is empty. However, the property of the S-matrix shown in (6) refers the S-elements to the center of a three-port junction containing the negligibly small lumped element network. Therefore if scattering measurements are made on only two ports and the three-port S-matrix constructed, errors due to the physical size of the device in relation to the wavelength will be introduced. For the active semiconductor region itself, such error is negligible for frequencies at which transistors are likely to operate in the near future, but the encapsulations are sufficiently large to produce errors. However, if future encapsulations are designed to provide access lines of characteristic admittance up to the semiconductor chip itself, then this source of error will be insignificant.

Appendix[1]

Proof of Scattering Matrix Result

The foregoing correspondence states that "it is readily seen that the rows and columns of $[S]$ (the scattering matrix in Anderson's notation) add to unity." Since the general proof of this does not appear in any of the texts and works consulted either on circuit theory or matrix algebra it is presented as follows.

If two matrices $[A]$ and $[B]$ are such that their rows and/or columns sum to a, b, respectively, then it is evident that the rows and/or columns of the matrix $[A+B]$ will sum to $a+b$ and less obvious (but true) that their product sums to the product ab. Let us check the product property for, say, the row sum. The row sum is as follows:

$$\sum_k [AB]_{ik} = \sum_k \sum_j A_{ij} B_{jk} = \sum_j A_{ij} \sum_k B_{jk} = \sum A_{ij} b = ab. \quad (8)$$

The column sum property is proved in the same manner.

Now the unit matrix $[1]$ obviously sums to one for both rows and columns. Using this fact, the product property and argument similar to (8) one can prove a useful property of the inverse matrix $[A^{-1}]$, by considering $[A][A^{-1}] = [1]$ for column sums and $[A^{-1}][A] = [1]$ for row sums. The property is this: the inverse of a matrix whose rows and/or columns sum to a is a matrix whose rows and/or columns sum to a^{-1}. Now the scattering matrix $[S]$ is given by

$$[S] = ([1] - [Y])([1] + [Y])^{-1} \quad (9)$$

and we are told that the normalized admittance matrix $[Y]$ has, by Kirchhoff's laws, rows and columns which sum to zero. The row and column sum of the scattering matrix $[S]$ is immediately $[(1-0)(1+0)^{-1} = 1]$ one.

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[1] The Appendix to this correspondence was written by T. W. Johnston, RCA Victor Plasma and Space Physics Research Laboratory, Montreal, Canada.

Transmission-Line Treatment of Waveguides Filled with a Moving Medium

To the author's knowledge, a theoretical study of guided waves in a moving isotropic medium was first investigated by Collier and Tai.[1] They derived the electromagnetic fields within a source-free region of a circular or rectangular waveguide by the method of vector potentials.

This correspondence discusses the problem of determining the vector fields produced by arbitrary electric and magnetic impressed currents in a uniform waveguide of arbitrary cross section filled with a dielectric medium, which moves down the waveguide with a constant velocity, by representing the fields in terms of a suitable set of vector mode functions.

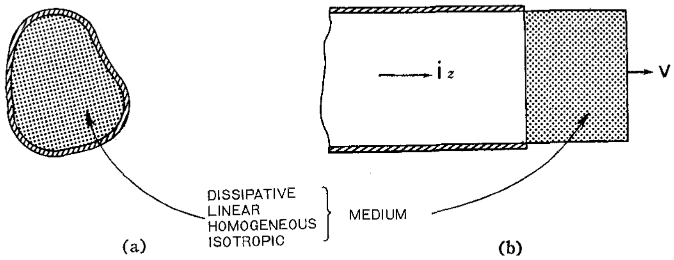

Fig. 1. Uniform waveguide of arbitrary cross section filled with a moving medium. (a) Cross sectional view. (b) Longitudinal view.

Let us assume that a homogeneous, isotropic, and lossy medium is moving with constant velocity $\boldsymbol{v} = v\boldsymbol{i}_z$, past an observer at rest with respect to the waveguide (see Fig. 1). As long as the velocity of the medium is much smaller than the velocity of light, the electromagnetic fields inside a waveguide, measured in the rest frame of the observer, can be determined by the following Maxwell-Minkowski equations:[2]

$$\begin{aligned} (\nabla - j\omega\boldsymbol{\Lambda}) \times \boldsymbol{E} &= -j\omega\mu\boldsymbol{H} - \boldsymbol{J}^* \\ (\nabla - j\omega\boldsymbol{\Lambda} - \mu\sigma\boldsymbol{v}) \times \boldsymbol{H} &= j\omega\hat{\epsilon}\boldsymbol{E} + \boldsymbol{J} \end{aligned} \quad (1)$$

where

$\boldsymbol{\Lambda} = (\epsilon\mu - \epsilon_0\mu_0)\boldsymbol{v} = \Lambda\boldsymbol{i}_z$, $\Lambda = (\epsilon\mu - \epsilon_0\mu_0)v$

$\hat{\epsilon} = \epsilon(1 - j\sigma/\omega\epsilon)$

ϵ_0, μ_0 = permittivity and permeability of free space

ϵ, μ, σ = permittivity, permeability, and conductivity of the medium at rest

$\boldsymbol{J}, \boldsymbol{J}^*$ = impressed electric and magnetic current densities

and the time variation of the fields has been assumed to be $e^{+j\omega t}$.

To obtain the solution of the foregoing inhomogeneous field equations, we first transform these equations into more familiar forms. This can be done by letting

$$\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_0 e^{+j\omega\Lambda z}, & \boldsymbol{H} &= \boldsymbol{H}_0 e^{+j\omega\Lambda z} \\ \boldsymbol{J} &= \boldsymbol{J}_0 e^{+j\omega\Lambda z}, & \boldsymbol{J}^* &= \boldsymbol{J}_0^* e^{+j\omega\Lambda z}. \end{aligned} \quad (2)$$

Substituting (2) into (1), we obtain the following equations for $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$:

$$\begin{aligned} \nabla \times \boldsymbol{E}_0 &= -j\omega\mu\boldsymbol{H}_0 - \boldsymbol{J}_0^* \\ (\nabla - \mu\sigma\boldsymbol{v}) \times \boldsymbol{H}_0 &= j\omega\hat{\epsilon}\boldsymbol{E}_0 + \boldsymbol{J}_0. \end{aligned} \quad (3)$$

Even if the medium involved is moving with constant velocity along the perfectly conducting guide walls, the electric and magnetic fields should satisfy the same boundary conditions as in the case of stationary media:[3]

$$\begin{aligned} \boldsymbol{n} \times \boldsymbol{E} &= 0 \\ \boldsymbol{n} \cdot \boldsymbol{H} &= 0 \end{aligned} \quad \text{or} \quad \begin{aligned} \boldsymbol{n} \times \boldsymbol{E}_0 &= 0 \\ \boldsymbol{n} \cdot \boldsymbol{H}_0 &= 0 \end{aligned} \quad (4)$$

where $\boldsymbol{n}$ denotes a unit vector normal to the guide walls. The electric and magnetic fields $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ which satisfy the inhomogeneous field equations (3) and subject to the boundary conditions (4), are then expressed in the following forms in terms of the vector mode functions:

$$\begin{aligned} \boldsymbol{E}_{0t} &= \sum_i [V_i^e(z)\boldsymbol{M}_i^e + V_i^m(z)\boldsymbol{M}_i^m] \\ \boldsymbol{H}_{0t} &= \sum_i [-I_i^e(z)\boldsymbol{N}_i^e + I_i^m(z)\boldsymbol{N}_i^m] \\ \boldsymbol{E}_{0z} &= \sum_i [(k_i^e)^2 V_{zi}(z)\boldsymbol{M}_{zi}] \\ \boldsymbol{H}_{0z} &= \sum_i [(k_i^m)^2 I_{zi}(z)\boldsymbol{N}_{zi}] \end{aligned} \quad (5)$$

where V_i^e, V_i^m, V_{zi} and I_i^e, I_i^m, I_{zi} are mode voltages and currents, respectively. The superscripts e and m denote the electric (or TM) modes and the magnetic (or TE) modes, respectively. Also, the subscripts t and z are employed to designate the transverse field components and the longitudinal ones, respectively.

The vector mode functions $\boldsymbol{M}_i^e$, $\boldsymbol{N}_i^e$, $\boldsymbol{M}_{zi}$ and $\boldsymbol{M}_i^m$, $\boldsymbol{N}_i^m$, $\boldsymbol{N}_{zi}$ are characterized by the following equations:

$$\begin{aligned} \boldsymbol{M}_i^e &= \nabla_t\Phi_i^e \quad , & \boldsymbol{M}_i^m &= \nabla_t\Phi_i^m \times \boldsymbol{i}_z \\ \boldsymbol{N}_i^e &= \nabla_t\Phi_i^e \times \boldsymbol{i}_z, & \boldsymbol{N}_i^m &= \nabla_t\Phi_i^m \\ \boldsymbol{M}_{zi} &= \Phi_i^e\boldsymbol{i}_z \quad , & \boldsymbol{N}_{zi} &= \Phi_i^m\boldsymbol{i}_z \end{aligned} \quad (6)$$

where the functions Φ_i^e and Φ_i^m are derived from the scalar eigenvalue problems

$$\begin{aligned} \nabla_t^2\Phi_i^e + (k_i^e)^2\Phi_i^e &= 0 \\ \nabla_t^2\Phi_i^m + (k_i^m)^2\Phi_i^m &= 0 \end{aligned} \quad (7)$$

subject to

$$\begin{aligned} \Phi_i^e &= 0 \\ \partial\Phi_i^m/\partial n &= 0 \end{aligned} \quad (8)$$

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[1] J. R. Collier, and C. T. Tai, "Guided waves in moving media," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-13, pp. 441–445, July 1965.

[2] C. T. Tai, "A study of electrodynamics of moving media," *Proc. IEEE*, vol. 52, pp. 685–689, June 1964.

[3] R. C. Costen, and D. Adamson, "Three-dimensional derivation of the electrodynamic jump conditions and momentum-energy laws at a moving boundary," *Proc. IEEE*, vol. 53, pp. 1181–1187, September 1965.

on the boundary. The vector mode functions possess the following orthogonality properties:

$$\iint \boldsymbol{M}_i^e\cdot\boldsymbol{M}_j^e dS = \iint \boldsymbol{N}_i^e\cdot\boldsymbol{N}_j^e dS = (k_i^e)^2\iint \Phi_i^e\Phi_j^e dS$$
$$\iint \boldsymbol{M}_i^m\cdot\boldsymbol{M}_j^m dS = \iint \boldsymbol{N}_i^m\cdot\boldsymbol{N}_j^m dS = (k_i^m)^2\iint \Phi_i^m\Phi_j^m dS = \begin{cases}1 & \text{for } i=j\\ 0 & \text{for } i\neq j\end{cases}$$
$$\iint \boldsymbol{M}_i^e\cdot\boldsymbol{M}_j^m dS = \iint \boldsymbol{N}_i^e\cdot\boldsymbol{N}_j^m dS = 0, \qquad \text{for all } i,j. \tag{9}$$

where the integrations are evaluated over the entire guide cross section.

From (3) and (5), together with the boundary conditions (4) and the orthogonality properties (9), we obtain:

$$\frac{dV_i^e}{dz} - (\alpha_i^e/j\omega\hat{\epsilon})I_i^e = (1/j\omega\hat{\epsilon})g_i^e(z)$$
$$\frac{dI_i^e}{dz} - \mu\sigma v I_i^e + j\omega\hat{\epsilon}V_i^e = -h_i^e(z) \tag{10}$$
$$\frac{dV_i^m}{dz} + j\omega\mu I_i^m = -h_i^m(z)$$
$$\frac{dI_i^m}{dz} - \mu\sigma v I_i^m - (\alpha_i^m/j\omega\mu)V_i^m = -(1/j\omega\mu)g_i^m(z) \tag{11}$$

$$I_i^e = -j\omega\hat{\epsilon}V_{zi} - \iint J_{0z}\Phi_i^e dS$$
$$V_i^m = -j\omega\mu I_{zi} - \iint J_{0z}^*\Phi_i^m dS. \tag{12}$$

In the foregoing equations, we have introduced the following notations:

$$\alpha_i^e = \hat{k}^2 - (k_i^e)^2, \quad \alpha_i^m = \hat{k}^2 - (k_i^m)^2$$
$$\hat{k}^2 = \omega^2\hat{\epsilon}\mu = \omega^2\epsilon\mu(1 - j\sigma/\omega\epsilon)$$
$$g_i^e(z) = j\omega\hat{\epsilon}\iint J_{0t}^*\cdot N_i^e dS - (k_i^e)^2\iint J_{0z}\Phi_i^e dS$$
$$g_i^m(z) = j\omega\mu\iint J_{0t}\cdot \boldsymbol{M}_i^m dS + (k_i^m)^2\iint J_{0z}^*\Phi_i^m dS$$
$$h_i^e(z) = \iint J_{0t}\cdot \boldsymbol{M}_i^e dS$$
$$h_i^m(z) = \iint J_{0t}^*\cdot \boldsymbol{N}_i^m dS. \tag{13}$$

Thus, the problem of determining the vector fields in a waveguide filled with a moving medium is reduced to that of solving an infinite set of inhomogeneous transmission-line equations (10) and (11). The complete solution of (10) and (11) is composed of a part due to sources at finite distance and a part due to sources at infinity; these correspond, respectively, to the particular solution and the complementary solution of the equations.

The particular solution of the inhomogeneous transmission-line equations (10) and (11) may be obtained by means of the technique of scalar Green's function as follows:

$$V_i^e(z) = \frac{1}{j\omega\hat{\epsilon}}\frac{1}{\gamma_+^e - \gamma_-^e}\int_{-\infty}^{z}[(\gamma_+^e)g_i^e(z') + (\alpha_i^e)h_i^e(z')]e^{\gamma_-^e(z-z')}dz'$$
$$+ \frac{1}{j\omega\hat{\epsilon}}\frac{1}{\gamma_+^e - \gamma_-^e}\int_{z}^{\infty}[(\gamma_-^e)g_i^e(z') + (\alpha_i^e)h_i^e(z')]e^{\gamma_+^e(z-z')}dz' \tag{14}$$
$$-I_i^e(z) = -\frac{1}{\gamma_+^e - \gamma_-^e}\int_{-\infty}^{z}[g_i^e(z') + (\gamma_-^e)h_i^e(z')]e^{\gamma_-^e(z-z')}dz'$$
$$- \frac{1}{\gamma_+^e - \gamma_-^e}\int_{z}^{\infty}[g_i^e(z') + (\gamma_+^e)h_i^e(z')]e^{\gamma_+^e(z-z')}dz' \tag{15}$$

where

$$\gamma_\pm^e = \frac{\mu\sigma v}{2} \pm \sqrt{\left(\frac{\mu\sigma v}{2}\right)^2 + [(k_i^e)^2 - \hat{k}^2]} \tag{16}$$

and

$$V_i^m(z) = -\frac{1}{\gamma_+^m - \gamma_-^m}\int_{-\infty}^{z}[g_i^m(z') + (\gamma_+^m)h_i^m(z')]e^{\gamma_-^m(z-z')}dz'$$
$$- \frac{1}{\gamma_+^m - \gamma_-^m}\int_{z}^{\infty}[g_i^m(z') + (\gamma_-^m)h_i^m(z')]e^{\gamma_+^m(z-z')}dz' \tag{17}$$

$$I_i^m(z) = \frac{1}{j\omega\mu}\frac{1}{\gamma_+^m - \gamma_-^m}\int_{-\infty}^{z}[(\gamma_-^m)g_i^m(z') + (\alpha_i^m)h_i^m(z')]e^{\gamma_-^m(z-z')}dz'$$
$$+ \frac{1}{j\omega\mu}\frac{1}{\gamma_+^m - \gamma_-^m}\int_{z}^{\infty}[(\gamma_+^m)g_i^m(z') + (\alpha_i^m)h_i^m(z')]e^{\gamma_+^m(z-z')}dz' \tag{18}$$

where

$$\gamma_\pm^m = \frac{\mu\sigma v}{2} \pm \sqrt{\left(\frac{\mu\sigma v}{2}\right)^2 + [(k_i^m)^2 - \hat{k}^2]}. \tag{19}$$

If the impressed sources are distributed within a finite region, $z_1 < z < z_2$, mode voltages V_i^e, for example, reduce to

$$V_i^e(z) = \begin{cases}\dfrac{1}{j\omega\hat{\epsilon}}\dfrac{1}{\gamma_+^e - \gamma_-^e}\displaystyle\int_{z_1}^{z_2}[(\gamma_+^e)g_i^e(z') + (\alpha_i^e)h_i^e(z')]e^{\gamma_-^e(z-z')}dz', & (z > z_2)\\[2ex] \dfrac{1}{j\omega\hat{\epsilon}}\dfrac{1}{\gamma_+^e - \gamma_-^e}\displaystyle\int_{z_1}^{z_2}[(\gamma_-^e)g_i^e(z') + (\alpha_i^e)h_i^e(z')]e^{\gamma_+^e(z-z')}dz', & (z < z_1).\end{cases} \tag{20}$$

$$E_t = \sum_i [Z_i^e A_i^e M_i^e e^{\Gamma_i^e z} + B_i^m M_i^m e^{\Gamma_i^m z}]$$
$$H_t = \sum_i [-A_i^e N_i^e e^{\Gamma_i^e z} + Y_i^m B_i^m N_i^m e^{\Gamma_i^m z}]$$
$$E_z = -(1/j\omega\hat{\epsilon}) \sum_i [A_i^e (k_i^e)^2 M_{zi} e^{\Gamma_i^e z}]$$
$$H_z = -(1/j\omega\mu) \sum_i [B_i^m (k_i^m)^2 N_{zi} e^{\Gamma_i^m z}] \qquad (27)$$

where

$$\Gamma_i^e = \pm j \sqrt{[\hat{k}^2 - (k_i^e)^2] - \left(\frac{\mu\sigma v}{2}\right)^2} + \left(j\omega\Lambda + \frac{\mu\sigma v}{2}\right)$$
$$\Gamma_i^m = \pm j \sqrt{[\hat{k}^2 - (k_i^m)^2] - \left(\frac{\mu\sigma v}{2}\right)^2} + \left(j\omega\Lambda + \frac{\mu\sigma v}{2}\right) \qquad (28)$$

and

$$Z_i^e = \frac{1}{Y_i^e} = \frac{1}{\omega\hat{\epsilon}}\left[\mp \sqrt{[\hat{k}^2 - (k_i^e)^2] - \left(\frac{\mu\sigma v}{2}\right)^2} - j\frac{\mu\sigma v}{2}\right]$$
$$Y_i^m = \frac{1}{Z_i^m} = \frac{1}{\omega\mu}\left[\mp \sqrt{[\hat{k}^2 - (k_i^m)^2] - \left(\frac{\mu\sigma v}{2}\right)^2} + j\frac{\mu\sigma v}{2}\right]. \qquad (29)$$

Thus, the electromagnetic fields produced by an arbitrary distribution of sources in a uniform waveguide of arbitrary cross section filled with a moving medium, can be determined from (2), (5), and (14)–(19).

Let us next obtain the complementary solutions for the mode voltages and currents, i.e., the solutions of (10) and (11) for the homogeneous case:

$$\frac{dV_i^e}{dz} - (\alpha_i^e/j\omega\hat{\epsilon})I_i^e = 0$$
$$\frac{dI_i^e}{dz} - \mu\sigma v I_i^e + j\omega\hat{\epsilon}V_i^e = 0 \qquad (21)$$
$$\frac{dV_i^m}{dz} + j\omega\mu I_i^m = 0$$
$$\frac{dI_i^m}{dz} - \mu\sigma v I_i^m - (\alpha_i^m/j\omega\mu)V_i^m = 0. \qquad (22)$$

To determine explicit solutions of (21) and (22), it is convenient to eliminate either V_i or I_i yielding the one dimensional equations:

$$\frac{d^2V_i}{dz^2} - \mu\sigma v\frac{dV_i}{dz} + \alpha_i V_i = 0$$

or

$$\frac{d^2I_i}{dz^2} - \mu\sigma v\frac{dI_i}{dz} + \alpha_i I_i = 0 \qquad (23)$$

where the superscript distinguishing the mode type has been omitted for simplicity, since the equations are of the same form for both modes. Thus, the complementary solutions are written in the following form:

$$I_i = A_i e^{\gamma_\pm z}, \quad V_i = B_i e^{\gamma_\pm z} \qquad (24)$$

where A_i and B_i are constants and $\gamma_\pm$ are given in (16) or (19). The characteristic impedances and admittances can be defined by

$$Z_i = \frac{V_i}{I_i}, \qquad Y_i = \frac{I_i}{V_i}. \qquad (25)$$

For the homogeneous case, (12) is reduced to

$$V_{zi} = -(I_i^e/j\omega\hat{\epsilon})$$
$$I_{zi} = -(V_i^m/j\omega\mu). \qquad (26)$$

From expressions (24)–(26), the electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ within a source-free region of a waveguide filled with a moving medium can be expressed as follows:

The above expressions include, as their special cases, the results obtained by Collier and Tai,[1] who have derived, by the method of vector potentials, the electromagnetic fields within a source-free region of a circular or rectangular waveguide.

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Synthesis of Particular Unit Real Functions of Reflection Coefficient

Physically realizable input reflection coefficients of circuits consisting of commensurable lossless transmission lines and resistors have been described in terms of the parameter $z = e^{-j[(f/f_0)\pi]}$ where f is frequency and f_0 the frequency at which a line is a quarter-wavelength long, resulting in a function of z, $\rho(z)$, introduced in Young[1] and there called unit real (ur) functions. Later, the definition of ur functions was extended, and a test procedure was given for them.[2] In the original paper, it was shown that for a given unit real function $\rho(z)$, with reference characteristic impedance Z_0, and satisfying $\rho(0)\rho(\infty) = 1$, a unit element

$$Z_1 = Z_0 \frac{1 + \rho(0)}{1 - \rho(0)}$$

could be removed leaving a reflection coefficient

$$\rho_1(z) = \frac{1}{2}\,\frac{\rho(z) - \rho(0)}{1 - \rho(z)\rho(0)} \qquad (1)$$

which would be a lower degree than $\rho(z)$ in both numerator and denominator and which would still be ur.

For a given $\rho(z)$ which satisfies $\rho(0)\rho(\infty) = 1$, there is no guarantee that $\rho_1(0)\rho_1(\infty)$ will necessarily also be equal to unity, and so on, and therefore that the process of removal of unit elements will necessarily end in a resistive termination. The problem to be discussed in this correspondence is the form that $\rho(z)$ must have so that it will be capable of realization by cascaded transmission lines and a resistive termination. From this discussion, a procedure which tests any $\rho(z)$ to see whether or not it has the required form will be derived.

Obviously, from the work previously referred to, it is necessary that $\rho(z)$ be ur and therefore this must be the first test in the procedure. This condition by itself is not sufficient. This is demonstrated by the fact that the input $\rho(z)$ of a cascade of any combination of unit elements, stubs and resistors, a general form not allowed for the restricted problem under consideration, is still ur. For the general form, both the magnitude of $\rho(+1)$, the $\lambda/2$ line condition, or the magnitude of $\rho(-1)$, the $\lambda/4$ line condition, will be equal to unity, whereas for the desired form of a cascade of unit elements terminated in a pure resistor $\rho(\pm 1)$ is a real number less than unity. Hence, a second necessary condition is that $|\rho(\pm 1)| \neq 1$. Let

$$\rho(z) = \frac{a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0}{b_n z^n + b_{n-1}z^{n-1} + \cdots + b_1 z + b_0} \qquad (2)$$

from which, for $\rho(0)\rho(\infty) = 1$,

$$\frac{a_0 a_n}{b_0 b_n} = 1. \qquad (3)$$

A special case of (3) occurs when

$$\frac{a_n}{b_n} = \frac{a_0}{b_0} = \pm 1 \qquad (4)$$

and if coefficients of (2) are made to conform to this condition and the resulting $\rho(z)$ is substituted into (1), it will be seen that there results a value of $\rho_1(z)$ for which the degree of the denominator is one greater than that of the numerator and for which therefore $\rho_1(0)\rho_1(\infty) \neq 1$. This means that in this case the process of reduction of degree of input reflection coefficient by removal of cascade lines cannot be continued further. The condition that $\rho(0) \neq \rho(\infty)$ therefore constitutes a third necessary condition.

If the successive removal of unit elements is investigated by repeated substitution of (2) into (1), the conditions being imposed at any kth stage that $\rho_k(0)\rho_k(\infty) = 1$, but $\rho_k(0) \neq \rho_k(\infty)$, it is easily found that if the original $\rho(z)$ is of order n in both numerator and denominator, it is necessary that the following relationships between the coefficients of $\rho(z)$ must be satisfied simultaneously:

$$\left.\begin{array}{ll} 1) & a_0 a_n = b_0 b_n \\ 2) & a_0 a_{n-1} + a_1 a_n = b_0 b_{n-1} = b_1 b_n \\ 3) & a_0 a_{n-2} + a_1 a_{n-1} + a_2 a_n = b_0 b_{n-2} \\ & \qquad + b_1 b_{n-1} + b_2 b_n \\ n) & a_0 a_1 + a_1 a_2 + \cdots + a_{n-1} a_n = b_0 b_1 \\ & \qquad + b_1 b_2 + \cdots + b_{n-1} b_n. \end{array}\right\} \qquad (5)$$

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[1] L. Young, "Unit-real functions in transmission-line circuit theory," *IRE Trans. on Circuit Theory*, vol. CT-7, pp. 247–250, September 1960.

[2] C. S. Gledhill, "Resistor transmission-line circuits," *Proc. IEE (London)*, vol. 112, p. 2046, November 1965.